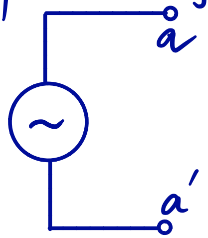
Three phase circuits

Modern power system transmits three phase-shifted sinusoidal currents through three parallel wires across long distances. From these three-phase (denoted 3ϕ) circuits, often a single phase supplies your home. However, in this course, we will only deal with 3ϕ generators supplying power to 3ϕ loads only.

Why 3ϕ? Why not 1ϕ, 2ϕ, 4ϕ, etc.?

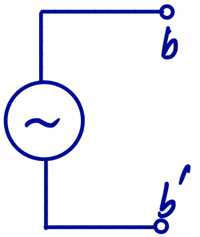
Three phase power generation has advantages that we will cover later in the course.

To explain 3ϕ circuits, consider 3 sinusoidal voltage sources that are phase-shifted by 120 degrees as follows.



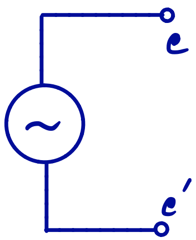
$$v_{aa'}(t) = v_0 \cos(\omega t + \theta_v)$$

$$\bar{V}_{aa'} = \frac{v_0}{\sqrt{2}} \angle \theta_v$$



$$v_{bb'}(t) = v_0 \cos(\omega t + \theta_v - 120^\circ)$$

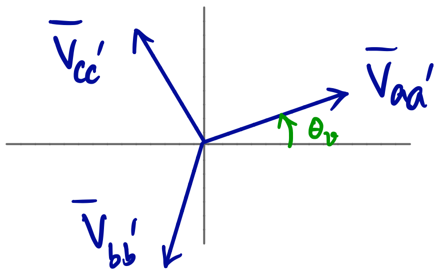
$$\bar{V}_{bb'} = \frac{v_0}{\sqrt{2}} \angle (\theta_v - 120^\circ)$$

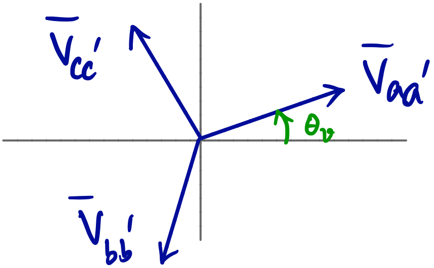


$$v_{cc'}(t) = v_0 \cos(\omega t + \theta_v + 120^\circ).$$

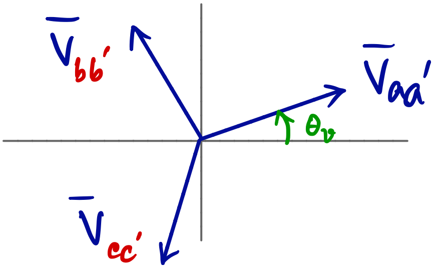
$$\bar{V}_{cc'} = \frac{v_0}{\sqrt{2}} \angle (\theta_v + 120^\circ)$$

Phasor diagram:





Such a sequence is called a positive (+ve) sequence.

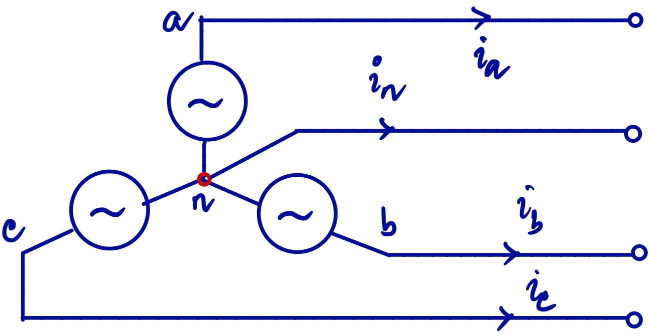


Such a sequence is called a negative (-ve) sequence.

<u>Claim:</u> $\bar{V}_{aa'} + \bar{V}_{bb'} + \bar{V}_{cc'} = 0.$

Verify it with elementary complex arithmetic.

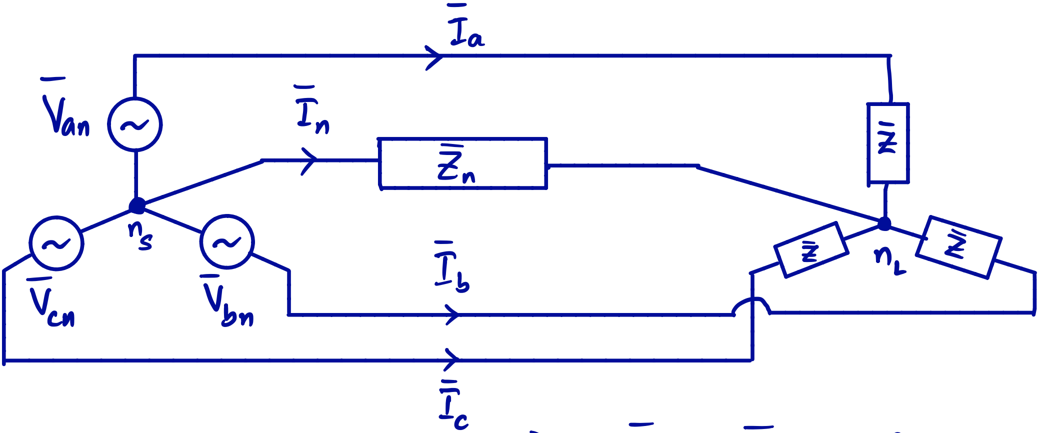
Let us interconnect a', b', c' terminals.



Terminal "n" is called neutral.

This interconnection is a "WYE" connected source.

Let us interconnect this "wye" connected source to a "wye" connected load as follows



- Claim 1: $\bar{I}_n = 0$, and $\bar{V}_{n_s} - \bar{V}_{n_L} = 0$.

Proof:

$$\bar{V}_{an} - \bar{I}_a \bar{Z} + \bar{I}_n \bar{Z}_n = 0.$$

$$\bar{V}_{bn} - \bar{I}_b \bar{Z} + \bar{I}_n \bar{Z}_n = 0.$$

$$\bar{V}_{cn} - \bar{I}_c \bar{Z} + \bar{I}_n \bar{Z}_n = 0$$

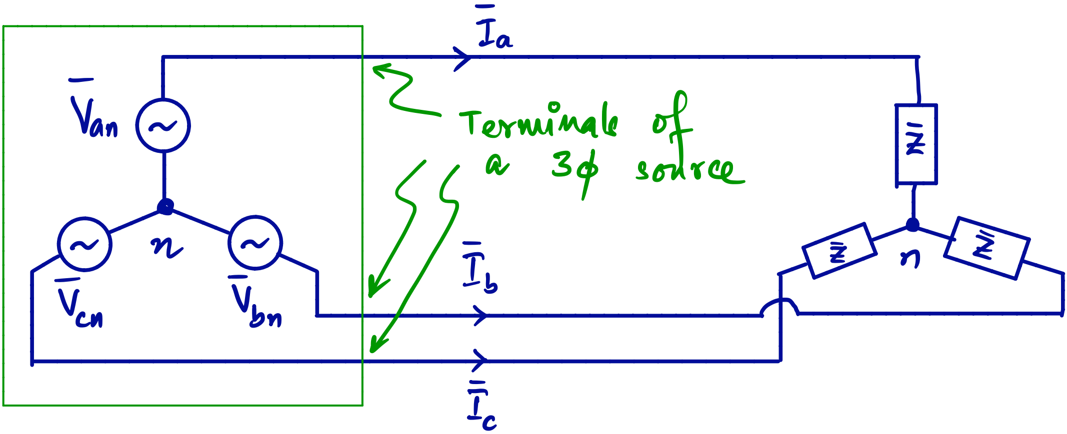
Add them together.

$$0 - \underbrace{\left(\bar{I}_a + \bar{I}_b + \bar{I}_c\right)}_{= -\bar{I}_n \text{ by Kirchhoff's current law.}} \bar{Z} + 3\bar{I}_n \bar{Z}_n = 0.$$

$$\Rightarrow \bar{I}_n \left(3\bar{Z}_n + \bar{Z}\right) = 0 \Rightarrow \bar{I}_n = 0 \text{ (why?).}$$

In this course, we will only consider "balanced" loads, i.e., each branch or phase of the load is identical. For balanced 3ϕ sources & loads, $\bar{I}_n = 0$

$\Rightarrow$ we can ignore the line interconnecting the neutral terminals.



<u>Definition</u>: The magnitude of voltage differences across the branches or phases of a 3ϕ-component is called phase voltage.

$$V_\phi = |\bar{V}_{an}| = |\bar{V}_{bn}| = |\bar{V}_{cn}|.$$

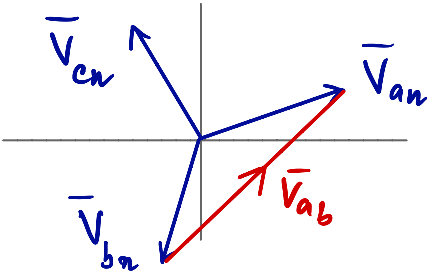
<u>Definition</u>: Magnitude of voltage differences between the terminals of a 3ϕ-component is called line-to-line or line voltage.

$$V_L = |\bar{V}_{ab}| = |\bar{V}_{bc}| = |\bar{V}_{cn}|.$$

- Claim: $V_L = \sqrt{3}\, V_\phi$.

Proof:

$$
\begin{aligned}
V_L &= |\bar{V}_{ab}| \\
&= |\bar{V}_{an} - \bar{V}_{bn}| \\
&= \left| \frac{v_0}{\sqrt{2}} \angle \theta_v - \frac{v_0}{\sqrt{2}} \angle(\theta_v - 120^\circ) \right| \\
&= \left| \frac{v_0}{\sqrt{2}} \angle \theta_v \left(1 - e^{j\omega(-120^\circ)} \right) \right| \\
&= \frac{v_0}{\sqrt{2}} \cdot \left| 1 - \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) \right| \\
&= V_\phi \left| \frac{3}{2} + j\frac{\sqrt{3}}{2} \right| = \sqrt{3} \cdot V_\phi .
\end{aligned}
$$

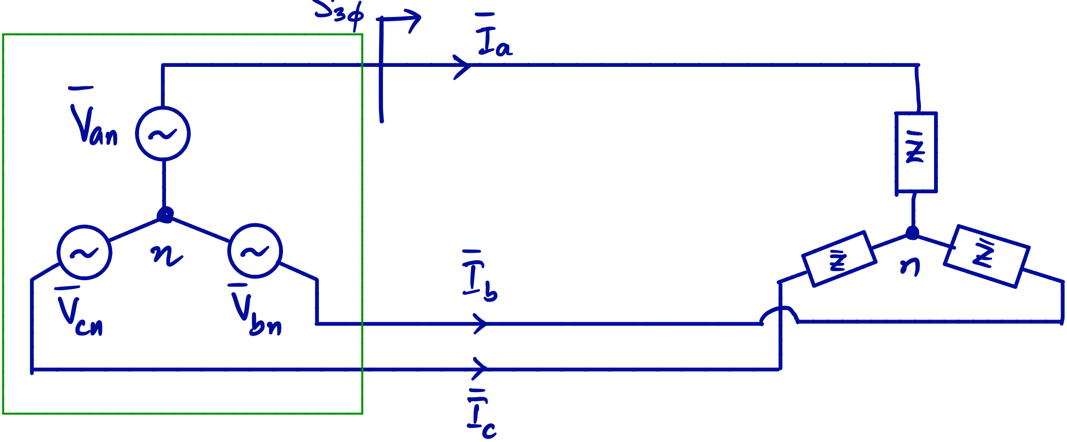


Phasor diagram showing phase and line voltage phasors.

<u>Definition</u>: Current magnitude on each branch of a 3ϕ component is called phase current. And, current magnitude flowing at the terminal is called line current.

For the wye-connected source, phase current (I_ϕ) equals line current (I_L), given by $|\bar{I}_a|$, i.e.,

$$I_\phi = I_L = |\bar{I}_a| = |\bar{I}_b| = |\bar{I}_c|.$$



- <u>Claim</u>: The 3ϕ complex power is given by $S_{3\phi} = \sqrt{3}\, V_L I_L \angle\theta$, where θ is the angle between $\bar{V}_{an}$ & $\bar{I}_a$.

<u>Proof</u>:

$$S_{3\phi} = \bar{V}_{an}\bar{I}_a^{*} + \bar{V}_{bn}\bar{I}_b^{*} + \bar{V}_{cn}\bar{I}_c^{*}.$$

Recall that phases b and c are phase shifted from a by $-120°$ and $+120°$, respectively.

$$\Rightarrow S_{3\phi} = \bar{V}_{an}\bar{I}_a^{*} + \left(\bar{V}_{bn}\cancel{\angle -120°}\right)\left(\bar{I}_a\cancel{\angle -120°}\right)^{*}$$
$$+ \left(\bar{V}_{an}\cancel{\angle +120°}\right)\left(\bar{I}_a\cancel{\angle +120°}\right)^{*}$$

$$= 3\,\bar{V}_{an}\,\bar{I}_a^*$$

$$= 3\left(|\bar{V}_{an}|\,\angle \bar{V}_{an}\right)\cdot\left(|\bar{I}_a|\cdot \angle(-\bar{I}_a)\right).$$

$$= 3\cdot|\bar{V}_{an}|\cdot|\bar{I}_a|\cdot \angle(\bar{V}_{an}-\bar{I}_a).$$

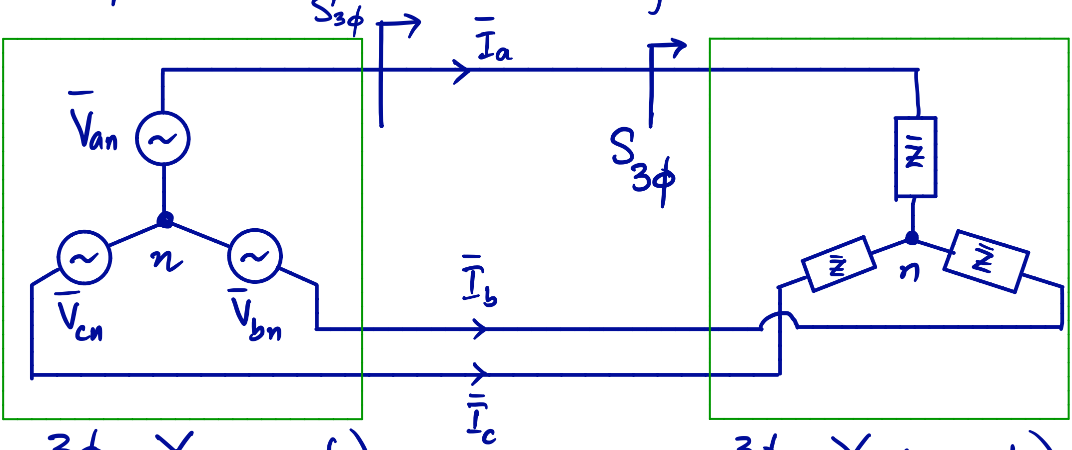
$$= 3\,V_\phi\,I_L\,\angle\theta$$

$$= \sqrt{3}\cdot V_L\,I_L\,\angle\theta.$$

$$\therefore\; P_{3\phi} = \sqrt{3}\,V_L\,I_L\cos\theta,$$

$$Q_{3\phi} = \sqrt{3}\,V_L\,I_L\sin\theta.$$

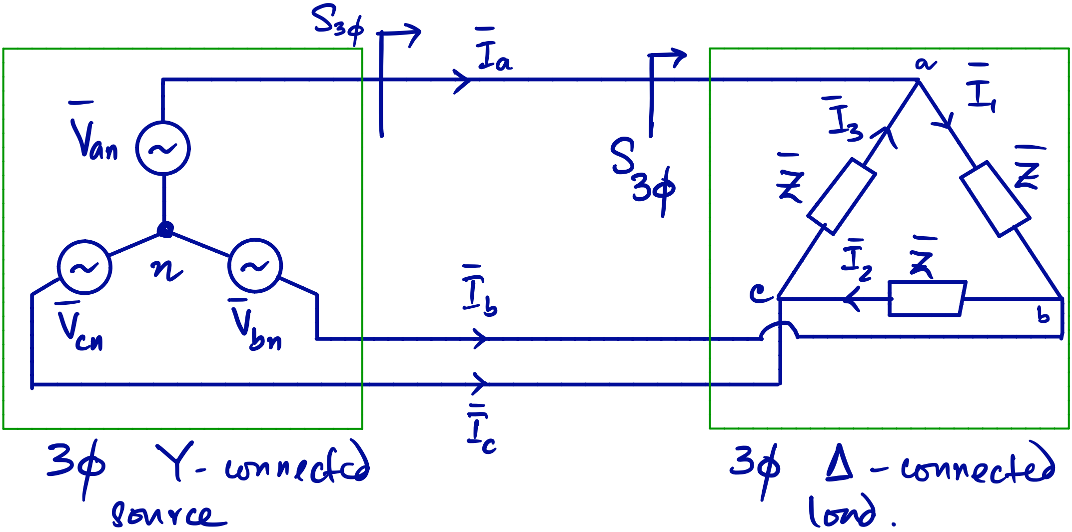
3ϕ real and reactive powers generated.



3ϕ Y-connected source

3ϕ Y-connected load.

Wye-connected (Y-connected) source & delta-connected (Δ-connected) load.



For the Δ-connected load,

phase voltage $V_\phi = |\bar{V}_{ab}| = |\bar{V}_{bc}| = |\bar{V}_{ca}|$.

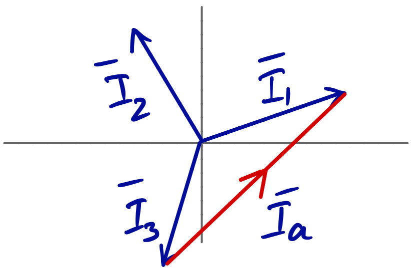
phase current $I_\phi = |\bar{I}_1| = |\bar{I}_2| = |\bar{I}_3|$.

line voltage $V_L = |\bar{V}_{ab}| = |\bar{V}_{bc}| = |\bar{V}_{ca}|$

line current $I_L = |\bar{I}_a| = |\bar{I}_b| = |\bar{I}_c|$.

$$\left.\begin{aligned} \bar{I}_a &= \bar{I}_1 - \bar{I}_3, \\ \bar{I}_b &= \bar{I}_2 - \bar{I}_1, \\ \bar{I}_c &= \bar{I}_3 - \bar{I}_2. \end{aligned}\right\} \text{From Kirchoff's current law.}$$

By symmetry, $\bar{I}_1, \bar{I}_2, \bar{I}_3$ are phasors with equal magnitudes, but phase-shifted by $120°$.



$|\bar{I}_1| = |\bar{I}_2| = |\bar{I}_3|$,

and $|\bar{I}_a| = \sqrt{3}\,|\bar{I}_1|$.

$\Rightarrow \quad I_L = \sqrt{3}\, I_\phi$.

Also, $V_L = V_\phi$.

<u>Claim</u>: The 3ϕ complex power delivered to the Δ-connected load is given by $S_{3\phi} = \sqrt{3}\, V_L I_L \cos\theta$, where θ is the angle between line voltages & currents, or phase voltages & currents.

$\bar{V}_{ab} = \bar{Z}\,\bar{I}_1$, $\bar{V}_{bc} = \bar{Z}\,\bar{I}_2$, $\bar{V}_{ca} = \bar{Z}\,\bar{I}_3$,

where $\bar{Z} = |\bar{Z}| . \angle\theta$.

Recall that

$$S_{3\phi} = 3 \cdot |\bar{V}_{an}| \cdot |\bar{I}_a| \cdot \measuredangle(\bar{V}_{an} - \bar{I}_a).$$

$$= \sqrt{3} \cdot |\bar{V}_{ab}| \cdot |\bar{I}_a| \cdot \measuredangle\theta$$

$$= \sqrt{3}\, V_L \cdot I_L \cdot \measuredangle\theta,$$

the same as a Y-connected source with a Δ-connected load.

Summary of relationships:

Y-component	Δ-component
$V_L = \sqrt{3}\, V_\phi$	$V_L = V_\phi$,
$I_L = I_\phi$	$I_L = \sqrt{3}\, I_\phi$,
$S_{3\phi} = \sqrt{3}\, V_L I_L \measuredangle\theta$	$S_{3\phi} = \sqrt{3}\, V_L I_L \measuredangle\theta$